Title: Using Technology to Review Parabolic Functions

Link to Outcomes:

- **Problem Solving** Students will demonstrate an understanding of parabolas through application.
- **Communication** Students will discuss (clarify and question) their written generalizations with other students.
- **Reasoning** Students will make and test their generalizations by creating more examples and graphing given parabolic functions without the use of technology.
- **Algebra** Students will explore the relation between the parabolic function $y = a(x - h)^2 + k$ and the patterns of their graphs.
- **Technology** Students will use Derive and/or graphics calculators.
- **Cooperation** Students will work in pairs to complete the assigned activities.
- **Connection** Students will use parabolic functions to solve real-life problems in sports (baseball) and electronics.

Brief Overview:

The parabola is assumed to be the path of a tossed object. In this lesson, students will review/explore the distinguishing features of the parabola by using technology to make generalizations that will reinforce what was previously learned.

Grade/Level:

Grades 10-12, Pre-Calculus

Duration/Length:

One to two class periods

Prerequisite Knowledge:

Students must have skills in using Derive or the graphics calculator. They should have a strong algebra background.

Objectives:

Student will

- graph quadratic equations on the graphics calculator or Derive.
- use the graphs to determine the result of changing the values of a and k in the equation $y = ax^2$.
- determine the result of changing the values of a and k in the equation $y = ax^2 + k$.
- determine the result of changing the values of a, k, and h in the equation $y = a(x - h)^2 + k$.
- sketch a parabolic equation given values of a, h, and k for the equation $y = a(x - h)^2 + k$.
- demonstrate an understanding of parabolic functions in an activity requiring application.

Materials/Resources/Printed Materials:

- Graphics calculator and/or computer (using Derive software)
- Student Activity Sheets

Development/Procedure:

The teacher will arrange students in groups of two. Each group will use a computer and/or graphics calculator to complete the given activities. Students will use a discovery approach to review and build on concepts introduced in earlier algebra courses. Students will follow instructions given on the activity sheets. They will graph parabolic functions on the computer and/or graphics calculator, sketch these graphs on activity sheets, and use these sketches to examine how changing values affect the graphs.

Evaluation:

Written assessment

Extension/Follow-up:

An exploration of the relationship between the vertex and maximum and minimum; parabolic equations that are not functions; changing any quadratic equation to the form $y = a(x - h)^2 + k$ by completing the square.

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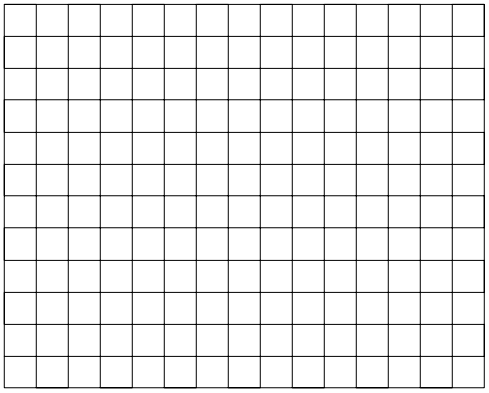
ACTIVITY 1
EXPLORING PARABOLIC FUNCTIONS OF THE FORM $y = ax^2$

A. Graph and sketch the graphs of the following functions.

$y = x^2$

$y = 2x^2$

$y = 4x^2$



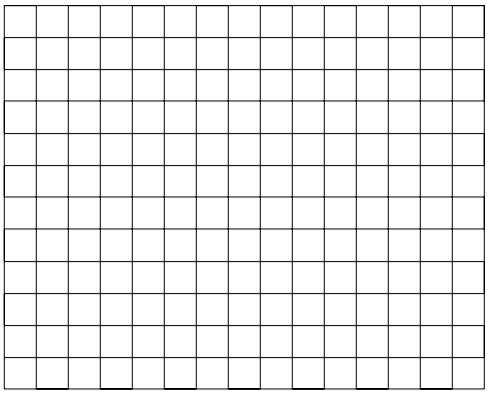
As *a* is increased, how is the graph affected?

_____

B. Graph and sketch the graphs of the following functions.

$y = -2x^2$

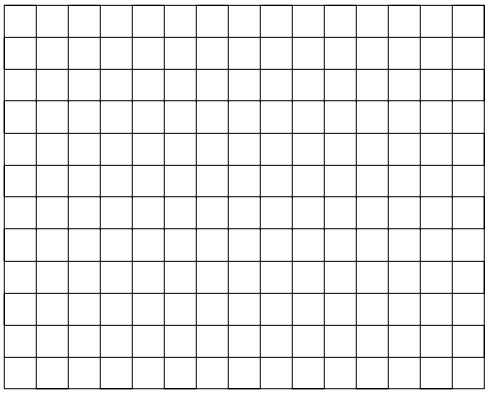
$y = -5x^2$



How do negative values of *a* affect the graph?

_____

C. Graph and sketch the graphs of the following functions.



$y = x^2$

$y = \frac{1}{2}x^2$

$y = \frac{1}{3}x^2$

$y = \frac{1}{4}x^2$

As a approaches 0, how is the graph affected?

_____

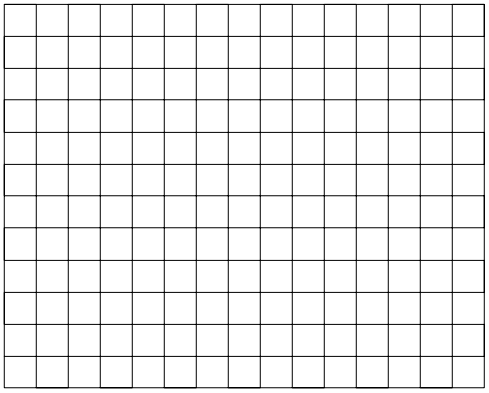
Predict the effect of changing a on the graph of $y = ax^2$.

_____

Check by graphing a few other examples.

ACTIVITY 2
EXPLORING PARABOLIC FUNCTIONS OF THE FORM $y = ax^2 + k$.

A. Graph and sketch the graphs of the following functions.



$y = x^2$

$y = x^2 + 2$

$y = x^2 + 3$

$y = x^2 - 4$

How does a change in k affect the graph?

_____

Predict the effect of changing both a and k on the graph of $y = ax^2 + k$.

_____

Check by graphing a few other examples.

ACTIVITY 3
EXPLORING PARABOLIC FUNCTIONS OF THE FORM $y = a(x - h)^2$

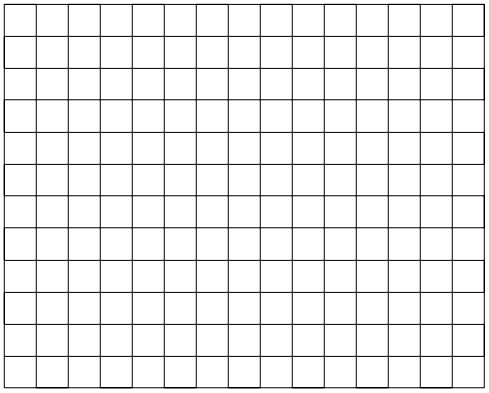
A. Graph and sketch the graphs of the following functions.

$y = x^2$

$y = (x - 2)^2$

$y = (x - 4)^2$

$y = (x + 3)^2$



As *h* is changed, how is the graph affected?

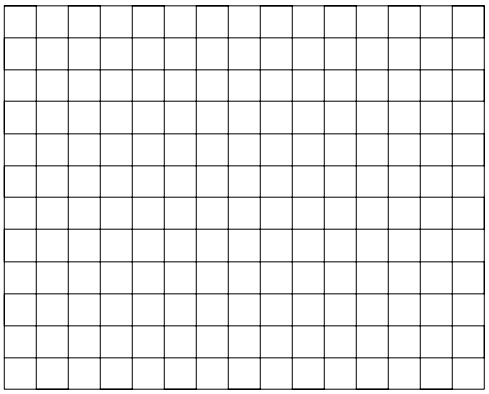
_____

B. Graph and sketch the graphs of the following functions.

$y = 2x^2$

$y = 2(x - 3)^2$

$y = -3(x + 4)^2$



As *a* and *h* are both changed, how is the graph affected?

_____

ACTIVITY 4
EXPLORING PARABOLIC FUNCTIONS OF THE FORM $y = a(x - h)^2 + k$

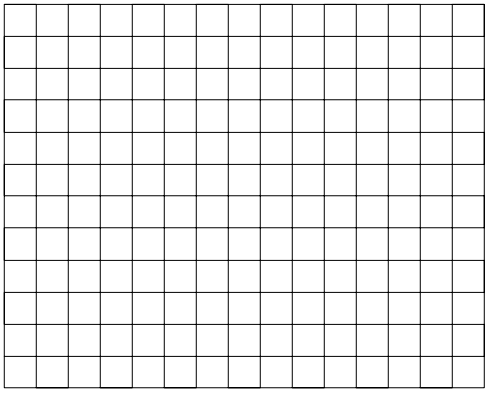
A. Graph and sketch the graphs of the following functions.

$y = (x - 3)^2 + 2$

$y = 2(x - 3)^2 + 2$

$y = 2(x + 4)^2 - 6$

$y = -\frac{1}{2}(x + 2)^2 - 6$

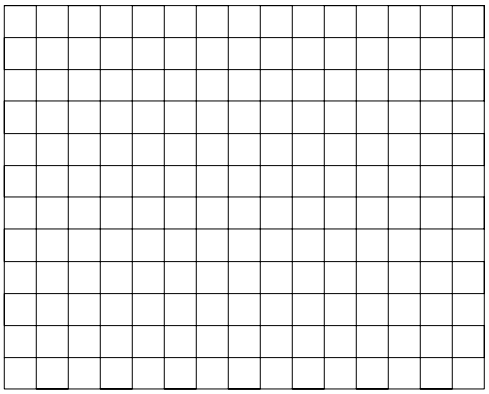


How do changes in *a*, *h*, and *k* respectively change the graph of the function?

_____

B. Without using a computer or calculator, sketch the graph of the following function.

$y = \frac{1}{3}(x - 6)^2 + 5$



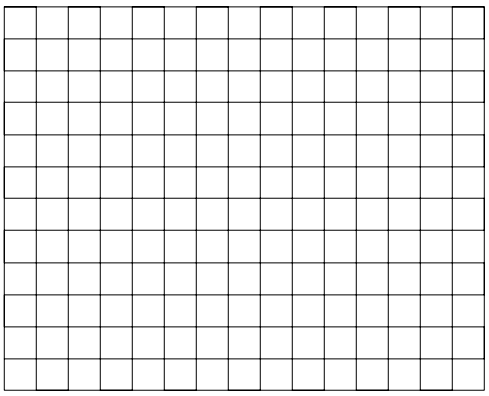
How is the point (6,5) significant in this graph?

_____

How will determining the coordinate (*h*,*k*) help us in graphing the function?

_____

C. Given $y = 3(x - 6)^2 - 8$, answer the following questions and proceed to sketch them without using a calculator or computer:



1. What is a?

_____

2. How does the graph turn?

_____

3. What is the vertex?

_____

4. Does this vertex represent a maximum or a minimum?

_____

5. Are there x-intercepts? If so, what are they?

_____

6. What is the y-intercept?

_____

Sketch the graph $y = 3(x - 6)^2 - 8$.

Graph $y = \frac{1}{2}(x + 4)^2 - 3$ and $y = 5(x + 3)^2 - 2$.

Check the graph by using a calculator or computer.

ASSESSMENT

Answer each question assuming that a, h, and k are real numbers and $a \neq 0$.

1. How does the graph of $y = (x - h)^2$ compare to the graph of $y = x^2$?

_____

2. How does the graph of $y = x^2 + k$ compare to the graph of $y = x^2$?

_____

3. How does the graph of $y = (x - h)^2 + k$ compare to the graph of $y = (x - h)^2$?

_____

4. How does the graph of $y = (x - h)^2 + k$ compare to the graph of $y = x^2 + k$?

_____

5. How does the graph of $y = (x - h)^2 + k$ compare to the graph of $y = x^2$?

_____

6. How does the graph of $y = ax^2$ compare to the graph of $y = x^2$?

_____

7. How does the graph of $y = ax^2$ compare to the graph of $y = ax^2 + k$?

_____

8. How does the graph of $y = a(x - h)^2 + k$ compare to the graph of $y = (x - h)^2 + k$?

_____

9. How does the graph of $y = a(x - h)^2 + k$ compare to the graph of $y = x^2$?

_____

Taken from Merrill Algebra 2 With Trigonometry by Macmillan/McGraw-Hill Publishing Company. (1992)

APPLICATION

In the finals of the softball tournament, Jenny hit a home run to win the game. The ball she hit, traveled in a path described by the function $f(x) = -0.004x^2 + x + 4$, where x represents the number of feet the ball has traveled from the plate and $f(x)$ represents the height in feet the ball traveled. Determine how many feet the ball has traveled from the batter when it reaches its highest point. Set up a table to solve this problem. Check the answer by graphing the function.

ANSWER KEY

Activity 1: The wording may vary for the answers.

A. The graph is narrower.

B. The graph turns downward, and as *a* decreases, the graph is narrower.

C. The graph is wider.

Prediction: The width of the graph is determined by *a*. As *a* increases, the graph narrows. If *a* is negative and as it approaches 0, the graph narrows.

Activity 2:

Change in *k*: *k* causes a vertical shift up or down.

Prediction: The value of *a* determines the width and *k* is the vertical shift.

Activity 3:

A. *h* causes a horizontal shift to the right or left.

B. *a* causes the graph to become wider or narrower, and *h* causes a horizontal shift to the right or left.

Activity 4:

A. *a* affects the width, *h* causes a horizontal shift, and *k* causes a vertical shift.

B. (6,5) is the minimum point (vertex) on the graph.

(*h*, *k*) is the vertex (minimum or maximum).

C. 1. 3
2. upward
3. (6,-8)
4. minimum
5. yes, approximately 4.5 and 7.6
6. 100

Assessment: The wording may vary for the answers.

1. $|h|$ units to the left or right.
2. $|k|$ units above or below.
3. $|k|$ units above or below.
4. $|h|$ units to the left or right.
5. $|h|$ units to the left or right and $|k|$ units above or below.
6. The graph would be narrower or wider.
7. The graph would be narrower or wider.
8. The graph would be narrower or wider.
9. $|h|$ units to the left or right, $|h|$ units above or below, narrower if $|a|<1$, wider if $|a|>1$, opens downward not upward if $a<0$.

Application:

125 feet